

Duality Symmetric Strings, Dilatons and $O(d,d)$ Effective Actions

David S. Berman¹ and Daniel C. Thompson²

Queen Mary College, University of London,
Department of Physics,
Mile End Road,
London, E1 4NS, England

Abstract

We calculate the background field equations for the T-duality symmetric string building on previous work by including the effect of the Dilaton up to two-loops. Inclusion of the Dilaton allows us to obtain the full beta functionals of the duality symmetric sigma model. We are able to interpret the result in terms of a dimensionally reduced $O(d,d)$ invariant target space effective action.

¹email: D.S.Berman@qmul.ac.uk

²email: D.C.Thompson@qmul.ac.uk

1 Introduction

Recently there has been much interest in the study of non-geometric backgrounds and in particular T-folds [1]. These are target spaces in which locally geometric patches are glued with transition functions taking values in the T-duality group. Although these T-folds are certainly a part of any string landscape they have proven to be surprisingly elusive to construct explicitly.

To understand string theory more easily on such target spaces it is natural to promote T-duality to be a manifest symmetry of the sigma model action [2]. One recent proposal to do just this is the ‘Doubled Formalism’ in which a target space with a T^d fibration is extended to one with a T^{2d} fibration [3, 4]. The additional d bosons in the doubled fibration are constrained so that they are identified with the T-dual. The sigma-model in this formalism has been shown to be equivalent to the standard sigma-model classically. Equivalence at the quantum level has been shown for the $d = 1$ case in the partition function [7, 9] and in canonical quantisation with Dirac brackets [8].

One important question on the quantum equivalence is to understand the beta functions. It is not at all obvious that these should be the same as the standard sigma model; they may receive interesting corrections from the T-dual. This was partially answered in [10] where the ultra-violet Weyl divergence coefficients of the doubled sigma-model were calculated. The geometric interpretation of these Weyl divergences was unclear from the point of view of a doubled target space. It was only after integrating out the additional T-dual bosons that the divergences were seen to be equivalent to those obtained for the standard sigma model.

We should ask what meaning the doubled-beta functions have in their own right without eliminating the dual coordinates. This is an interesting question since one might hope to find T-fold solutions more readily by trying to find doubled target spaces which satisfy the vanishing of the doubled-beta functions. However, as we show in this paper, the correct geometric interpretation of the doubled-beta functions is, perhaps surprisingly, not in terms of either the doubled or non-doubled target spaces.

In fact, the beta functions yield the field equations for the dimensional reduction of the standard target space theory. This dimensionally reduced theory has manifest $O(d, d)$ symmetry. This connection between the manifest $O(d, d)$ theory and T-duality manifest world sheets has been discussed previously in [11].

In this letter we shall begin by very briefly reviewing the formalism and approach of [10]. We will then extend these results to include the contribution of the Dilaton to first order in α' . After extracting the full beta functions (related to the Weyl divergence by demanding target space diffeomorphism invariance) we finally demonstrate their target space interpretation.

1.1 Formalism and Review of Results

The doubled formalism [1, 3, 4] is an alternate description of string theory on target spaces that are locally T^d bundles, with fibre coordinates X^i , over a base N with coordinates Y^a . The fibre is doubled to be a T^{2d} with $2d$ coordinates \mathbb{X}^A . The doubled sigma model then has Lagrangian³

$$\mathcal{L} = \frac{1}{4} H_{AB}(Y) d\mathbb{X}^A \wedge *d\mathbb{X}^B + \mathcal{L}(Y) \quad (1)$$

where $\mathcal{L}(Y)$ is the standard Lagrangian for a string on the base and $H(Y)$ is a metric on the fibre which is assumed to depend only on the base coordinates⁴. One may choose a frame where the metric H has an $O(d, d)/O(d) \times O(d)$ coset form as follows:

$$H_{AB}(Y) = \begin{pmatrix} h - bh^{-1}b & bh^{-1} \\ -h^{-1}b & h^{-1} \end{pmatrix}. \quad (2)$$

h and b are the target space metric and two-form field on the fibre of the un-doubled space. In this frame $\mathbb{X}^A = (X^i, \tilde{X}_i)$ with $\{\tilde{X}_i\}$ the coordinates on the T-dual torus.

³The complete formalism also introduces a 1-form connection for the fibration which we set to zero here. There is also a topological term which plays no role in our discussion although it is vital to obtaining the equivalence of the doubled to the non-doubled partition functions [7].

⁴Our conventions are that the worldsheet signature is $(+, -)$, $\partial_{\pm} = \partial_0 \pm \partial_1$, $\epsilon_{01} = 1 = \epsilon^{01}$ and for convenience we have dropped an overall factor of 2π . The factor of $\frac{1}{4}$ in (1) is half the usual normalisation and is required to make contact with the standard sigma model.

To eliminate the extra degrees of freedom, the doubled sigma model is supplemented with a set of worldsheet constraints

$$d\mathbb{X}^A = L^{AB} H_{BC} * d\mathbb{X}^C, \quad (3)$$

where the L is an $O(d, d)$ invariant metric such that $HL^{-1}H = L$. In the basis where H is given by (2),

$$L_{AB} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}. \quad (4)$$

We may introduce vielbeins to change to a basis where $H = \text{diag}(\mathbb{1}, \mathbb{1})$, $L = \text{diag}(\mathbb{1}, -\mathbb{1})$ and the constraint (3) is seen to be a chirality constraint on the bosons. In [10] this constraint was incorporated into the action using the method of Pasti, Sorokin and Tonin [12]. The resulting action, after gauge fixing the PST symmetry, looks like two copies of the Floreanini-Jackiw action [13]. In the basis of (2) the action we obtain is

$$S = \frac{1}{2} \int d^2\sigma \left[-\mathcal{G}_{\alpha\beta} \partial_1 X^\alpha \partial_1 X^\beta + \mathcal{L}_{\alpha\beta} \partial_1 X^\alpha \partial_0 X^\beta + \mathcal{K}_{\alpha\beta} \partial_0 X^\alpha \partial_0 X^\beta \right], \quad (5)$$

where $X^\alpha = (\mathbb{X}^A, Y^a)$ and

$$\mathcal{G}_{\alpha\beta} = \begin{pmatrix} H_{AB} & 0 \\ 0 & g_{ab} \end{pmatrix}, \mathcal{L}_{\alpha\beta} = \begin{pmatrix} L_{AB} & 0 \\ 0 & 0 \end{pmatrix}, \mathcal{K}_{\alpha\beta} = \begin{pmatrix} 0 & 0 \\ 0 & g_{ab} \end{pmatrix}, \quad (6)$$

in which g_{ab} is the standard sigma model metric for the base⁵. This action is a generalisation of Tseytlin's duality symmetric formalism [5, 6]. The equations of motion on the fibre integrate correctly to give the constraint (3) and hence the string wave equation.

⁵Notation: Calligraphic objects with Greek indices refer to the total doubled space; uppercase Roman objects with uppercase Roman indices refer to the doubled fibre; lowercase Roman objects correspond to the non-doubled picture with early alphabet lowercase Roman indices denoting the base and late indices denoting the non-doubled fibre. Hatted objects are those constructed out of the base metric only.

1.2 UV Divergences

In [10] the ultra-violet divergences in the doubled formalism were studied by performing a background field expansion [14] of the action (5). The fields X^α were written as the sum of a classical piece X_{cl}^α and a quantum fluctuation ξ^α .⁶ One expands the exponential of the action to quadratic order and takes Wick contractions of ξ to determine the one-loop ultra-violet or Weyl divergence. Since the action (5) is not manifestly worldsheet Lorentz covariant extra care there are some subtleties and we refer the reader to [10] for more details.

The result found in [10] was that the $\mathcal{L}\partial_1 X \partial_0 X$ term produced no divergence but that metric term $\mathcal{G}\partial_1 X \partial_1 X$ term did produce divergences with a coefficient which we denote by $\mathcal{W}_{\alpha\beta}$. These differed from the doubled space Ricci tensor (which is the geometric quantity one would naively have expected) and are given by

$$\mathcal{W}_{\alpha\beta} = \mathcal{R}_{\alpha\beta} + \mathcal{S}_{\alpha\beta}, \quad (7)$$

where $\mathcal{R}_{\alpha\beta}$ is the doubled space Ricci tensor and

$$\mathcal{S}_{\alpha\beta} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{8}\text{tr}(\partial_a H \partial_b H^{-1}) \end{pmatrix}. \quad (8)$$

In component form, we have on the base

$$\mathcal{W}_{ab} = \hat{R}_{ab} + \frac{1}{8}\partial_a H_{AB}\partial_b H^{AB}, \quad (9)$$

where \hat{R}_{ab} is the Ricci tensor constructed from the base metric g alone. And on the fibre

$$\mathcal{W}_{AB} = -\frac{1}{2}\partial^2 H_{AB} + \frac{1}{2}((\partial_a H)H^{-1}(\partial^a H))_{AB} + \frac{1}{2}\Gamma^t_{ab}g^{ab}\partial_t H_{AB}. \quad (10)$$

The divergence differs from the doubled space Ricci tensor by an extra factor of 1/2 in the term on the base containing the doubled fibre metric H . This is essential to make contact with the standard sigma-model in [10], it compensates for a doubled counting coming from doubling the fibre. This difference however makes it very hard to see a geometric interpretation of the divergence in the doubled picture. We shall return to address this later but first let us examine how the Dilaton changes the picture.

⁶Precisely, ξ is the tangent vector of the geodesic between the classical and quantum values.

2 Inclusion of the Dilaton

The doubled Dilaton Φ is related to the standard Dilaton ϕ by

$$\Phi(Y) = \phi(Y) + \frac{1}{2} \ln \det h \quad (11)$$

and is included into the string action (even in the doubled formalism) with a standard Fradkin-Tseytlin term⁷

$$S_{dil} = \frac{1}{8\pi} \int d^2\sigma \Phi(Y) R^{(2)}. \quad (12)$$

We emphasize that this term is an order of α' up on the rest of the doubled string action. We should remark that in the conformal gauge (which we have used to do the background field expansion) the Dilaton decouples from the theory.

2.1 Generalised Conformal Invariance

We can understand how the Dilaton enters into the metric Weyl divergence in a slightly indirect way [19]. Consider starting with a sigma model with no Dilaton term. To demand that the sigma model is Weyl anomaly free we don't actually need that $\mathcal{W}_{\alpha\beta}$ is identically zero. Since the sigma model should not be affected by on-shell target space field transformation induced by $X^\alpha \rightarrow X^\alpha + V^\alpha$ we only need that the Weyl divergence is zero up to such redefinition. Therefore we demand that the full beta function vanishes, that is:

$$\beta_{\alpha\beta} = \mathcal{W}_{\alpha\beta} - \mathcal{D}_\alpha V_\beta = 0. \quad (13)$$

As explained in [19] we should restrict V_β to be the derivative of a scalar which we are led to associate with the Dilaton. Hence

$$\beta_{\alpha\beta} = \mathcal{R}_{\alpha\beta} + \mathcal{S}_{\alpha\beta} - \mathcal{D}_\alpha \mathcal{D}_\beta \Phi = 0. \quad (14)$$

⁷Note the normalisation of $\frac{1}{8\pi}$ which is for later convenience. A different normalisation would require amending the relation between Φ and ϕ .

We express this on the base and fibre and obtain:

$$\beta_{ab} = \hat{R}_{ab} + \frac{1}{8} \partial_a H_{AB} \partial_b H^{AB} - \hat{\nabla}_a \hat{\nabla}_b \Phi, \quad (15)$$

$$\beta_{AB} = -\frac{1}{2} \hat{\nabla}^2 H_{AB} + \frac{1}{2} \left((\hat{\nabla}_a H) H^{-1} (\hat{\nabla}^a H) \right)_{AB} - \frac{1}{2} \hat{\nabla}_a H_{AB} \hat{\nabla}^a \Phi. \quad (16)$$

We have chosen to express these with hatted covariant derivatives constructed out of the base metric g_{ab} and the reader should bear in mind that these objects are actually blind to indices on the fibre e.g.

$$\hat{\nabla}_a H_{AB} = \partial_a H_{AB}, \quad \hat{\nabla}^2 H_{AB} = \partial_a \partial^a H_{AB} + \hat{\Gamma}_{ab}^a \partial^b H_{AB}.$$

With this notation we are tempted to think of fibre indices as not really labeling target space coordinates but labeling moduli fields contained in H . In other words it seems like these beta-functionals may have a geometric interpretation in a target space that has been dimensionally reduced so that only the base coordinates remain. We will see later that this is indeed a sensible interpretation.

2.2 Dilaton Beta Function

Now let us consider the Dilaton beta function. The leading order part will be the familiar

$$\beta^\Phi = \frac{26 - D}{6} + O(\alpha') \quad (17)$$

where D is the dimension of the non-doubled target space. The reason for this is seen quite clearly by counting the central charge c of the doubled theory; if we double d coordinates we have $D + d$ bosons however $2d$ of them are chiral and only count half in the central charge. We thus expect the central charge to remain equal to the standard target space dimension. The factor of 26 arises from the determinant that arise from integration over world sheet metrics [16]. From here on we shall work in the critical dimension.

The next order in α' contribution is more complicated. There are two sources of contributions, namely those which are 1-loop and arise from the background field

expansion of S_{dil} to quadratic order and those which are 2-loops and arise from the expansion of the action (5).

Let us first evaluate the one loop contribution. Since the Dilaton action is the same as for the standard string we can read off the 1-loop result that

$$\beta^\Phi = \frac{1}{2}\alpha' (-2\mathcal{D}_\alpha \mathcal{D}^\alpha \Phi - (D\Phi)^2 + \text{2-loops}) \quad (18)$$

$$= \frac{1}{2}\alpha' (-2\hat{\nabla}_a \hat{\nabla}^a \Phi - (\hat{\nabla}\Phi)^2 + \text{2-loops}) . \quad (19)$$

We have used that $\Gamma_{Ia}^I = \text{tr} H^{-1} \partial_a H = 0$ to get to the second line.

To evaluate the two loop contribution we need to background field expand (5) to third and fourth order. The relevant terms are those that do not directly couple to the classical fields ∂X^α and whose contractions have a power counting of p^{-2} . At third order we find

$$S^{(3)} = \frac{1}{2} \int \mathcal{L}_{\alpha\beta;\gamma} \xi^\gamma D_0 \xi^\alpha D_1 \xi^\beta + \mathcal{K}_{\alpha\beta;\gamma} \xi^\gamma D_0 \xi^\alpha D_0 \xi^\beta + \dots , \quad (20)$$

and at fourth order

$$\begin{aligned} S^{(4)} &= -\frac{1}{6} \int \mathcal{R}_{\alpha\sigma\delta\beta} D_1 \xi^\alpha D_1 \xi^\beta \xi^\sigma \xi^\delta \\ &+ \frac{1}{48} \int [12\mathcal{L}_{\alpha\beta;\sigma\delta} + 4(\mathcal{L}_{\alpha\rho} \mathcal{R}_{\sigma\delta}^\rho + \mathcal{L}_{\beta\rho} \mathcal{R}_{\sigma\delta}^\rho)] D_0 \xi^\alpha D_1 \xi^\beta \xi^\sigma \xi^\delta \\ &+ \frac{1}{48} \int [12\mathcal{K}_{\alpha\beta;\sigma\delta} + 8\mathcal{K}_{\alpha\rho} \mathcal{R}_{\sigma\delta}^\rho] D_0 \xi^\alpha D_0 \xi^\beta \xi^\sigma \xi^\delta + \dots , \end{aligned} \quad (21)$$

where the dots signify contributions not relevant for the Dilaton.

So to calculate the quantum effective action we will need to calculate the wick contractions of the exponential of the action. That is, we will have to consider $\langle (S^{(3)})^2 \rangle$ and $\langle S^{(4)} \rangle$. One could find the correct propagator contractions by considerations similar to those in the appendix of [10]. It would, of course, be a matter of some considerable detail to calculate all the diagrams needed. However we can quickly see by taking a few test contractions of $\mathcal{R}_{\alpha\beta\sigma\delta}$ with $\mathcal{G}^{\alpha\beta}$, $\mathcal{L}^{\alpha\beta}$ and $\mathcal{K}^{\alpha\beta}$ that the result, which must be a scalar, will be given in terms of two underlying objects; the base Ricci scalar \hat{R} and two-derivative contractions of the fibre coset metric H .

So with constants A and B undetermined we will have

$$\beta^\Phi = \frac{\alpha'}{2} \left(-2\hat{\nabla}_a \hat{\nabla}^a \Phi - (\hat{\nabla}\Phi)^2 + A\hat{R} + B \left(\hat{\nabla}_a H^{-1} \hat{\nabla}^a H \right) \right). \quad (22)$$

2.3 Integrability Condition

We shall use an indirect way to determine these coefficients. For the standard string the vanishing of the metric beta function and the Bianchi identity imply that the divergence of the metric beta function is equal to the gradient of some scalar. This scalar is identified as the Dilaton beta function. We show that with a little extra work the same is true for the doubled case, that is

$$\mathcal{D}^\alpha \beta(\mathcal{G})_{\alpha\beta} = \mathcal{D}_\beta \beta(\Phi) \quad (23)$$

It is straight forward to see that

$$\mathcal{D}^\alpha \beta(\mathcal{G})_{\alpha\beta} = \mathcal{D}^\alpha (\mathcal{R}_{\alpha\beta} + \mathcal{S}_{\alpha\beta} - \mathcal{D}_\beta \mathcal{D}_\alpha \Phi) \quad (24)$$

$$= \frac{1}{2} \mathcal{D}_\beta (\mathcal{R} - 2\mathcal{D}^2 \Phi - (\mathcal{D}\Phi)^2) + \mathcal{S}_{\alpha\beta} \mathcal{D}^\alpha \Phi + \mathcal{D}^\alpha \mathcal{S}_{\alpha\beta}. \quad (25)$$

It is also easy to see that on the fibre (when $\beta = B$) both sides of (23) are zero by the assumption that no fields depend on the fibre coordinates. So we will need to evaluate this expression on the base (when $\beta = b$). We find

$$\begin{aligned} \mathcal{D}^\alpha \beta_{\alpha\beta} \Big|_{\beta=b} &= \hat{\nabla}_b \left(\frac{\hat{R}}{2} + \frac{1}{8} \text{tr}(\hat{\nabla}_a H \hat{\nabla}^a H^{-1}) - \hat{\nabla}^2 \Phi - \frac{1}{2} (\hat{\nabla}\Phi)^2 \right) \\ &\quad + (D^\alpha \mathcal{S}_{\alpha\beta} + \mathcal{S}_{\alpha\beta} D^\alpha \Phi) \Big|_{\beta=b}. \end{aligned} \quad (26)$$

To proceed we are going to need a suitable Bianchi-like identity for $\mathcal{S}_{\alpha\beta}$ to allow us to pull out a total derivative. We can find such an expression by consider the base components of the doubled space Bianchi identity,

$$D^\alpha \mathcal{R}_{\alpha\beta} \Big|_{\beta=b} = \frac{1}{2} D_\beta \mathcal{R} \Big|_{\beta=b}. \quad (27)$$

On the right hand side one finds

$$\frac{1}{2}\hat{\nabla}_b\mathcal{R} = \hat{\nabla}_b\left(\frac{1}{2}\hat{R} + \frac{1}{8}\text{tr}(\hat{\nabla}_a H^{-1}\hat{\nabla}^a H)\right), \quad (28)$$

whilst on the left one finds with a bit of manipulation

$$\begin{aligned} D^\alpha\mathcal{R}_{\alpha\beta}\Big|_{\beta=b} &= \hat{\nabla}^a\hat{R}_{ab} + \frac{1}{4}\text{tr}(\hat{\nabla}_a H H^{-1}\hat{\nabla}_b H H^{-1}\hat{\nabla}^a H H^{-1}) \\ &\quad - \frac{1}{4}\text{tr}(\hat{\nabla}^a H H^{-1}\hat{\nabla}_a\hat{\nabla}_b H H^{-1}). \end{aligned} \quad (29)$$

After equating these last two expressions (and using the Bianchi identity of the base Ricci tensor) we have an identity that allows us to see that

$$\begin{aligned} \frac{1}{2}\hat{\nabla}_b\text{tr}(\hat{\nabla}_a H^{-1}\hat{\nabla}^a H) &= \text{tr}(\hat{\nabla}_a H H^{-1}\hat{\nabla}_b H H^{-1}\hat{\nabla}^a H H^{-1}) \\ &\quad - \text{tr}(\hat{\nabla}^a H H^{-1}\hat{\nabla}_a\hat{\nabla}_b H H^{-1}). \end{aligned} \quad (30)$$

We now look at the \mathcal{S} terms in (26). For the first we have

$$D^\alpha\mathcal{S}_{\alpha\beta}\Big|_{\beta=b} = -\frac{1}{8}\hat{\nabla}^a\text{tr}(\hat{\nabla}_a H\hat{\nabla}_b H^{-1}) \quad (31)$$

$$\begin{aligned} &= -\frac{1}{8}\text{tr}(\hat{\nabla}^2 H\hat{\nabla}_b H^{-1}) - \frac{1}{4}\text{tr}(\hat{\nabla}_a H H^{-1}\hat{\nabla}_b H H^{-1}\hat{\nabla}_a H H^{-1}) \\ &\quad + \frac{1}{8}\text{tr}(\hat{\nabla}^a H H^{-1}\hat{\nabla}_a\hat{\nabla}_b H H^{-1}). \end{aligned} \quad (32)$$

For the second we find

$$\mathcal{S}_{\alpha\beta}D^\alpha\Phi\Big|_{\beta=b} = -\frac{1}{8}\text{tr}(\hat{\nabla}_b H^{-1}\hat{\nabla}_a H)\hat{\nabla}^a\Phi = -\frac{1}{8}(\hat{\nabla}_b H^{IJ})\hat{\nabla}_a H_{IJ}\hat{\nabla}^a\Phi. \quad (33)$$

We can now use the vanishing of the fibre components of the doubled metric beta function (16) to swap the Dilaton for some more terms involving H . We find

$$\mathcal{S}_{\alpha\beta}D^\alpha\Phi\Big|_{\beta=b} = \frac{1}{8}\text{tr}(\hat{\nabla}_b H^{-1}\hat{\nabla}^2 H) + \frac{1}{8}\text{tr}(\hat{\nabla}_a H H^{-1}\hat{\nabla}_b H H^{-1}\hat{\nabla}_a H H^{-1}). \quad (34)$$

Upon invoking the identity (30) we find that

$$(D^\alpha\mathcal{S}_{\alpha\beta} + \mathcal{S}_{\alpha\beta}D^\alpha\Phi)\Big|_{\beta=b} = -\frac{1}{16}\hat{\nabla}^b\text{tr}(\hat{\nabla}_a H\hat{\nabla}^a H^{-1}). \quad (35)$$

Thus from the integrability condition (23) we determine the Dilaton beta function to be

$$\beta^\Phi = \frac{\alpha'}{2} \left(-2\hat{\nabla}_a \hat{\nabla}^a \Phi - (\hat{\nabla} \Phi)^2 + \hat{R} + \frac{1}{8} \left(\hat{\nabla}_a H^{-1} \hat{\nabla}^a H \right) \right). \quad (36)$$

3 Target space interpretation

For the standard string a result of fundamental importance is that the beta functions can be connected to the field equations of a target space theory [17]. The identification is schematically given by

$$\beta^G - \frac{1}{\alpha'} G \beta^\phi \sim \delta_G S, \quad (37)$$

$$\frac{1}{\alpha'} \beta^\phi \sim \delta_\phi S, \quad (38)$$

$$\beta^{matter} \sim \delta_{matter} S, \quad (39)$$

where S is the action for the target space theory. Given that we have seen that the beta functions appear geometric in a dimensionally reduced target space it is naturally to guess that the correct target space theory is the dimensional reduction of the standard bosonic string target space theory. We now show that this is indeed the case.

Starting with the d=26 bosonic space time action,

$$S_{26} = \int d^{26}x \sqrt{-G} e^\phi \left\{ R(G) + (\nabla \phi)^2 - \frac{1}{12} H^2 \right\} \quad (40)$$

we reduce on T^d with the relevant ansatz (in this case a simplified diagonal reduction) given by

$$G = \begin{pmatrix} h(Y) & 0 \\ 0 & g(Y) \end{pmatrix} \quad B = \begin{pmatrix} b(Y) & 0 \\ 0 & 0 \end{pmatrix} \quad \phi(Y) = \Phi(Y) - \frac{1}{2} \ln \det h. \quad (41)$$

With this ansatz we see that the T-dual invariant (i.e. doubled) Dilaton Φ emerges in the reduced action since

$$\sqrt{-G} e^\phi = \sqrt{-g} \sqrt{h} e^\phi = \sqrt{-g} e^{\phi + \frac{1}{2} \ln \det h} = \sqrt{-g} e^\Phi. \quad (42)$$

This expression also helps understand the choice of normalisation of (12). The standard result is a low energy effective action which displays manifest $O(d, d)$ symmetry e.g. [11, 21]

$$S_{26-d} = \text{vol}(\mathcal{M}^d) \int d^{26-d}y \sqrt{-g} e^\Phi \left\{ \hat{R}(g) + \left(\hat{\nabla} \Phi \right)^2 + \frac{1}{8} \text{tr} \left(L \hat{\nabla}_a H L \hat{\nabla}^a H \right) \right\}. \quad (43)$$

The fields arising from the internal components of the metric and B-field are thought of as moduli in that they parametrise the vacuum of the dimensionally reduced theory. They have been organised in an $O(d, d)$ invariant way by placing them in coset metric H_{IJ} .

We can see that the reduced action doesn't 'remember' which T-dual compactification it arose from. In this sense it is natural to expect a clear linkage with the doubled formalism. In performing this dimensional reduction we assumed that the fields had no dependence on the internal coordinates. This assumption doesn't contradict the general aim of understanding T-folds since a non-trivial fibration over the base is still allowed.

From the variation with respect to the metric we find Einstein's equation:

$$0 = \hat{R}_{ab} - \frac{1}{2} g_{ab} \hat{R} - T_{ab}, \quad (44)$$

where the stress-energy is given as

$$\begin{aligned} T_{ab} = & \hat{\nabla}_a \hat{\nabla}_b \Phi - g_{ab} \hat{\nabla}^2 \Phi - \frac{1}{2} g_{ab} \left(\hat{\nabla} \Phi \right)^2 \\ & - \frac{1}{8} \left(\text{tr} \left(\hat{\nabla}_a H^{-1} \hat{\nabla}_b H \right) - \frac{1}{2} g_{ab} \text{tr} \left(\hat{\nabla}_a H^{-1} \hat{\nabla}^a H \right) \right). \end{aligned} \quad (45)$$

Varying with respect to the fields H

$$0 = \hat{\nabla} \left(e^\Phi H^{-1} \hat{\nabla} H \right) \quad (46)$$

$$= e^\Phi H^{IJ} \left(\hat{\nabla}^2 H_{IJ} + \hat{\nabla}_a \Phi \hat{\nabla}^a H_{IJ} - \left(\hat{\nabla}_a H H^{-1} \hat{\nabla}^a H \right)_{IJ} \right), \quad (47)$$

and from the Dilaton

$$0 = -2 \hat{\nabla}^2 \Phi - \left(\hat{\nabla} \Phi \right)^2 + \hat{R} + \frac{1}{8} \text{tr} \left(\hat{\nabla} H^{-1} \hat{\nabla} H \right). \quad (48)$$

Immediately we see that fibre components of the doubled metric beta function β_{IJ} , given by (16), are proportional to the field equations for H_{IJ} (46), the doubled Dilaton beta function β^Φ is proportional to the field equation for Φ (48) and the identification rules (37, 38, 39) are satisfied with the base beta function β_{ab} playing the role of the β^G in (37) .

4 Conclusions

We conclude by making a few remarks about potential generalisations. In general the dimensional reduction above includes a $U(1)^{2d}$ gauge field coming from isometries of the T^d and transformations of the B-field on the fibre. It would be nice to shown that these fields arise when one considers a connection in the doubled-fibration. However, in this case it becomes more subtle to find the constrained action akin to (5). It seems that the PST procedure would result in off-diagonal elements in both \mathcal{G} and \mathcal{L} . This would significantly increase the computational burden in calculating the Weyl divergences.

To summarise, we have shown that the full beta-functions of the T-duality symmetric string led to a $O(d,d)$ invariant dimensionally reduced target space theory. This was perhaps to be expected though the utility of this paper is that the detailed relationship between the string in the doubled geometry and the equations of motion of the background are now made manifest. One hopes this may ultimately lead to a better understanding of how T-folds may work.

Note Added When this paper was in preparation two papers appeared concerning interesting aspects of T-folds and doubled geometry [22].

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